Uphill solitary waves in granular flows

E. Martínez,[1] C. Pérez-Penichet,[1] O. Sotolongo-Costa,[1] O. Ramos,[2] K. J. Måløy,[2] S. Douady,[3] and E. Altshuler[4,*]

[1]*"Henri Poincaré" Group of Complex Systems, Physics Faculty, University of Havana, 10400 Havana, Cuba*
[2]*Physics Department, University of Oslo, Blindern, N-0316 Oslo, Norway*
[3]*Laboratoire Matire et Systèmes Complexes (MSC), Universitè Paris 7, Denis Diderot/CNRS, CC 7056 75251 Paris Cedex 05, France*
[4]*"Henri Poincaré" Group of Complex Systems and Superconductivity Laboratory, Physics Faculty-IMRE, University of Havana,*
10400 Havana, Cuba

We have experimentally observed uphill solitary waves in the surface flow on a granular material. A heap is constructed by injecting sand between two vertical glass plates separated by a distance much larger than the average grain size, with an open boundary. As the heap reaches the open boundary, solitary fluctuations appear on the flowing layer and move "up the hill" (i.e., against the direction of the flow). We explain the phenomenon in the context of stop-and-go traffic models.

The rich dynamics of granular matter—studied for centuries by engineers—has attracted much attention from the physics community since the early 1990s [1–3]. Granular flows, for example, have concentrated intense interest due to their relevance to natural avalanches and industrial processes and also because they make liquidlike and solidlike behaviors coexist. They have been theoretically described based on the existence of two phases: the rolling (or flowing) one and the static one. Such an idea has been cast into *ad hoc* phenomenological equations [4–6], into a Saint-Venant hydrodynamic approach conveniently modified to take into account the particularities of granular matter [7,8], and eventually by defining an order parameter characterizing the local state of the system [9,10]. Finally, granular flows have been described by "microscopic" equations based on the Newtonian motion of individual grains submitted to gravity, shocks, and trapping events [11,12]. Laboratory research, on the other hand, has revealed a whole jungle of flow patterns in inclined plane experiments [13–18], sandpiles [19], tilted pipes [20,21], and other arrangements [23].

We report here solitonlike uphill waves when simply pouring sand on the surface of a heap formed between two vertical glass plates—i.e., the spontaneous appearance of bumplike instabilities in the flowing layer that propagate uphill, contrary to the flow of sand, as shown in Fig. 1.

We used sand with a high content of silicon oxide and an average grain size of 100 μm from Santa Teresa (Pinar del Río, Cuba) [19]. The sand was poured into a cell consisting of a horizontal base and a vertical wall sandwiched between two square glass plates with inner surfaces separated by a distance w in the range from 0.3 to 2 cm (Fig. 2). The lengths of the base and the vertical wall were approximately $d \approx 22$ cm. The sand was poured into the cell near the vertical wall using funnels with several hole diameters in order to obtain different flux values. As the sand was poured into the cell, a heap grew until it reached the open boundary, where the grains were allowed to fall freely (Fig. 2). Digital videos were taken using a high-speed video camera photron FAST-

CAM Ultima-APX model 120 K in the range from 50 to 4000 fps, with a resolution of 1024 × 1024 pixels.

Figure 1 contains results from experiments with w =0.6 cm and a flux per unit of width Q =1.1 cm^2/s. Figure 1(a) is a picture taken from a video recorded at 50 fps and shows a close-up of a section of the surface at the central region of the heap, where our basic findings are easily identified: "bump-shaped" instabilities appearing at random places of the surface move "up the hill," as indicated by the white arrows. These bumps could travel either alone or in tandem. They could be related to previous observations on inclined plane configurations, where regularly spaced uphill waves have been reported [17,18], but our instabilities do not appear when the sand is poured on a plane inclined at the average angle of repose with wooden, velvet, or sandpaper surfaces. In our experiments, bumps maintain their "identity" within variable time windows (as large as 7–10 s) and then flatten out. The shape persistence is illustrated in Fig. 1(b), which displays a sequence of profiles of a single bump as it travels upward along a distance of 10 cm. The profiles were constructed by linking with a line the places where a maximum brightness gradient is observed in the original pictures.
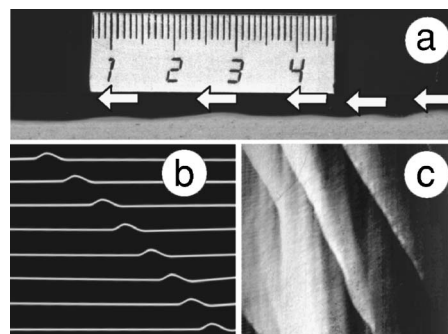

FIG. 1. Basic experimental findings. (a) Uphill bumps moving in tandem (identified by arrows) contrary to the flow of sand. (b) Profile of one bump at different instants of time as it travels uphill. In the picture, x grows to the right and t grows upward, spanning 10 cm and 5 s, respectively. (c) Spatial-temporal diagram of bump dynamics, where x and t grow in the same directions as in (b), but spanning 16 cm and 4 s, respectively.

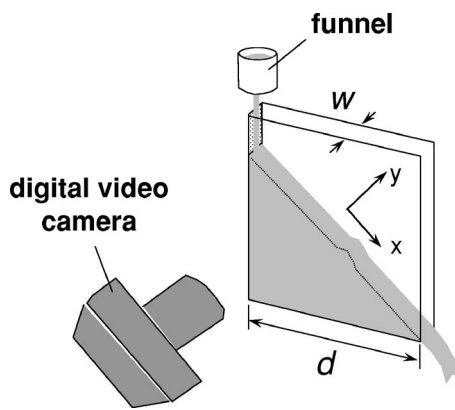*Electronic address: ea@infomed.sld.cu

FIG. 2. Simplified diagram of the experimental setup.

Figure 1(c) shows a spatial-temporal diagram of the bump dynamics. A row of pixels was taken at the unperturbed free surface (in a simplistic view, it follows the dotted line at h_0 in Fig. 5 below). The spatial-temporal diagram consists in a stack of these rows taken at different times. In the diagram, air zones look darker and sand zones look whiter. From the picture four shock waves can be clearly seen, moving upward with a well defined velocity (which is inversely proportional to the slope of the lines in the diagram). On the left, the appearance of a new bump can be seen, first increasing in height before starting suddenly to propagate.

As many phenomena in granular matter, the uphill waves depend, to some extent, on the way the heap is prepared and show some degree of "memory." When the heap is prepared by filling up the cell from an empty state, uphill bumps nucleate at random places near the center of the heap, and, within a few seconds, they typically appear near the open boundary. If the experiment is stopped and restarted in such conditions, even after a long time, solitons reappear immediately. That happens even if a layer of sand less than 1 mm thick is removed from the surface of the heap. If a thicker layer is removed, one has to wait a few seconds to observe them. This is probably due to the formation of a "compactified" layer during the flow that is necessary for the formation of these "jammed" bumps. Another important observation is the aging of the material, which reminds us of that observed in "singing sands" [23]: if the same sample is used many times to obtain bumps (which is easily the case when large fluxes are involved), a moment is reached where bumps are no longer obtained. This can be associated with some kind of layer on the grain surface which is removed or modified due to intergrain shocks. Finally, interaction between bumps have been observed, but these are very rare events.

The average speed of the uphill motion of the bumps as a function of the input flux per unit width, Q, was estimated from the analysis of the motion of several instabilities using spatial-temporal diagrams analogous to that shown in Fig. 1(c). Figure 3 presents the dependence of the average uphill speed versus input flux for different separations between the vertical walls, w (two flux values were examined for w =2 cm, but are not included in Fig. 3). For each value of w, we also report data from experiments where the roughness of one of the walls has been increased by gluing a 200-mesh sheet of sandpaper on it. The solid lines correspond to
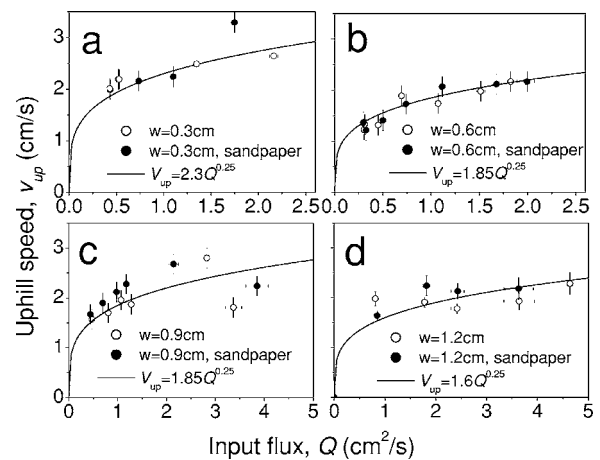

FIG. 3. Uphill speeds of bumps measured as a function of input flux for (a) w=3 mm, (b) 6 mm, (c) 9 mm, and (d) 12 mm. Each curve was measured with and without sandpaper glued to one of the vertical walls.

$v_{up} \sim Q^{1/4}$, which will be discussed below. The presence of a sheet of sandpaper on one wall has no measurable influence on v_{up} for each value of w (but it slightly increased the average angle of repose for some widths). This suggests that the uphill bumps are not connected to the friction with the walls, but with geometrical confinement. Additionally, we did not observe a significant dependence of the value of the uphill speed on w for a given value of Q within our experimental range $0.3 \text{ cm} \leq w \leq 2.0 \text{ cm}$.

To examine in detail the internal dynamics of a solitary wave, we took a sequence of pictures of one of them from a very short distance, at a rate of 4000 fps, one of them being illustrated in Fig. 4(a). Figure 4(b) contains a spatial-temporal diagram based on 1550 such pictures, separated by 2-ms intervals. The slicing was taken using the line seen in Fig. 4(a), and it spans approximately 24 mm along the horizontal axis and nearly 3 s along the vertical axis. Then, the diagram contains information during almost the full passage of the bump from right to left in Fig. 4(a). Four-well defined regions can be identified in Fig. 4(b). In region A, the bump has not entered the measurement area from the right and we see the grains flowing from left to right at a speed that can be estimated as a few mm/s. In region B, the "attack front" of the solitary wave is hit by the grains, resulting in a zone where they suddenly acquire a great speed (it can be thought as a jet of liquid hitting an obstacle). In region C, the grains are static under the bump, which is passing over them from right to left. Region D shows how the particles slowly reincorporate into the flowing layer after (most) of the bump has passed over them.

Exchange of particles between the static and fluid phases has been considered in several models of surface flows since the 1990s [4]. Based on the evidence provided by Fig. 4(b), we believe that our bumps form when a fluctuation implies an extra deposition of grains from the flowing layer on the static one. The resulting bumps will then propagate against the flow through an "stop-and-go" mechanism resembling that observed in traffic dynamics [25]. However, here the usual traffic models should be modified to take into account the free surface.
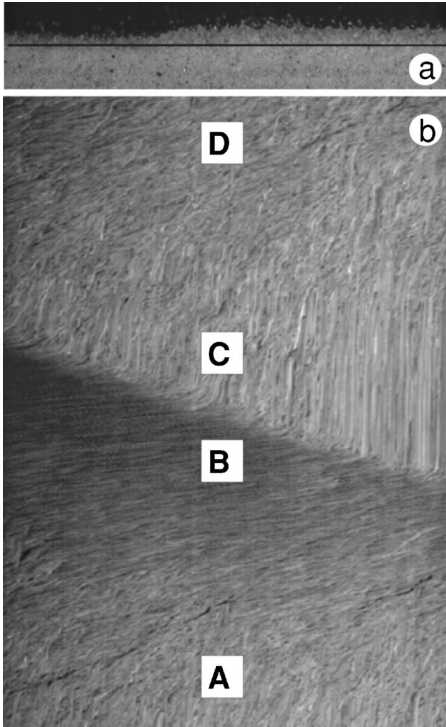
FIG. 4. Detailed dynamics of a solitary wave. (a) Sample picture of a single bump moving uphill. (b) Spatial-temporal diagram of the bump, taken along the horizontal line shown in (a). In the diagram, x grows to the right and t grows upward, spanning for 24 mm and 2 s, respectively.

The overall situation can be pictured as a solitary wave crawling uphill underneath a shallow stream of sand flowing downhill. In fact, the stop-and-go traffic model has been mapped into the "classical" Kortweg-de Vries (KdV) equation after a few approximations, resulting in solitonlike solutions [26]. In a "shallow water" scenario, the speed of a long-wave soliton can be estimated as $v_s \sim \sqrt{gh_0}$ where g and h_0 are the acceleration of gravity and the depth of the unperturbed stream, respectively [27] (in our case, the length of the perturbation is ~ 1 cm and $h_0 \sim 0.1$ cm, so the long-wave approximation holds). If one assumes the well-accepted result that the depth of the flowing layer is proportional to $Q^{1/2}$ [24], we then get $v_s \sim Q^{1/4}$, which follows the experimental result shown in Fig. 3.

Now, we propose a simple argument to show that the physical nature of granular flows allows the appearance of uphill solitary perturbations. We assume that our system can be described through modified Saint-Venant equations corresponding to mass and momentum conservation [7,8] which read, in a reference frame parallel to the average free surface (see Fig. 5),

$$\partial_t \zeta + \partial_x (\Gamma H^2/2) = 0, \tag{1}$$

$$\partial_t H + \partial_x (\Gamma H^2/2) = \frac{g}{\Gamma}[\tan \theta - \mu(H)], \tag{2}$$
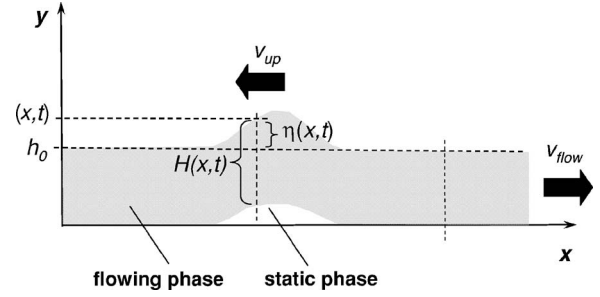

FIG. 5. Diagram showing the neighborhood of a "bump" which includes some parameters of the Saint-Venant model described in the text.

where ζ is the height of the free surface, H is the thickness of the flowing layer, Γ is the velocity gradient transversal to the flowing layer (assumed constant), $\mu(H)$ is the friction acting on the layer, and $\tan \theta = -\partial_x \zeta$ is the slope of the free surface. Then, $q = \Gamma H^2/2$ evolves with H due to an erosion and deposition process, like the one observed in our experiments.

It can be shown that solitary-wave solutions can be obtained from this equations if appropriate $\mu(H)$ dependences are assumed. To get the "textbook" KdV equation [27], for example, one has to assume that $\mu(H)$ has the general form $\mu = \alpha + (\beta + \frac{\gamma}{H})(h_0^2 - H^2)\sqrt{\delta - \varepsilon(h_0^2 - H^2)}$ where the values of the constants α, β, γ, δ, and ε can be expressed in terms of experimental parameters v_{up}, v_{flow}, h_0, and g and a characteristic length $L_0 = 3h_0$. Then, Eqs. (1) and (2) can be transformed into

$$h_0^2 v_{flow} \eta'' + \frac{5}{9}(v_{up} - v_{flow})\eta + 4\frac{v_{flow}}{h_0}\eta^2 = 0. \tag{3}$$

In Eq. (3), $\eta = \zeta - h_0$ and $\eta'' = \frac{d^2 \eta}{d\varphi^2}$, where $\varphi = x + v_{up}t$. If we take the realistic values $v_{flow} = \frac{1}{2}\Gamma h_0 \approx \sqrt{gh_0}$ and $v_{up} = \frac{1}{5}v_{flow}$, one gets the particular solution $\zeta = h_0 + (h_0/6)\mathrm{sech}^2[(x + v_{up}t)/3h_0]$, which describes a bell-shaped soliton moving upward. We remark, though, that other $\mu(H)$ choices are able to produce "non-KdV" solitary solutions when introduced in Eq. (2), reinforcing the importance of the specific features of the sand.

The phenomena presented here have been found in one type of sand that also produces sandpiles through "revolving rivers" [19,22]. We have tried a number of other sands, with negative results. In spite of the fact that we do not observe uphill bumps in inclined planes with wooden, velvet, or sandpaper surfaces, we have measured the stability diagram [14,28] for this sand in an incline with fixed bottom with a velvet surface in order to compare its properties with other sands through a well-established experiment. The diagram (which will be published elsewhere) was determined for wide ($w = 12$ cm) and narrow ($w = 0.6$ cm) channels, both showing uncommon large separations between the "stop" and "start" branches of the curves. However, we are unable to correlate "microscopic" features of this sand (chemical composition, grain shape, etc.) to its ability to show uphill bumps.

In summary, we have observed solitonlike instabilities in a flow of sand established on a heap, moving against the direction of the flow. The phenomenon can be understood in the light of stop-and-go traffic arguments. While the dependence of the friction coefficient on the depth of the flowing layer seems an important ingredient, the detailed "microscopic" mechanism that makes a certain sand able to show uphill bumps remains a mystery.

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